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LETTER TO THE EDITOR

# Comments on the Schrödinger equation with $\delta'$-interaction in one dimension

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**Abstract.** The boundary conditions at the singular point for the Schrödinger equation with $\delta'$-interaction in one dimension are proposed. It is also shown that the boundary conditions adopted by Albeverio *et al* are irrelevant to the $\delta'$-interaction.

In the study of the quantized Davey–Stewartson I system [1], as $N=2$ (two particles), we encountered the Schrödinger equation

$$-\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}+c\delta'(x)\psi=E\psi \tag{1}$$

where $c$ is the coupling constant, and $\delta'(x)$ is the derivative of the $\delta$-function. The same equation also appeared in [2–4]; however, it was dealt with incorrectly, i.e., the boundary conditions for it in [2–4] at the singular point are irrelevant to the $\delta'$-interaction.

The appropriate boundary conditions can be derived as follows. Integrating (1) from $0^-$ to $0^+$ and noting that $\delta'(x)\psi(x)=\delta'(x)\psi(0)-\delta(x)\psi'(0)$, we obtain $\psi'(0^+)-\psi'(0^-)=-c\psi'(0)$. Integrating (1) twice, from $-L<0$ to $x$ and from $0^-$ to $0^+$ respectively, we obtain $\psi(0^+)-\psi(0^-)=c\psi(0)$. These two relations for $\psi(0)$, $\psi(0^\pm)$, $\psi'(0)$ and $\psi'(0^\pm)$ are necessary conditions of a wavefunction $\psi$ being the solution of (1). As usual, we require $\psi$ to be continuous at $x=0$, then $\psi(0^+)=\psi(0^-)=\psi(0)=0$. So (1) can be replaced by

$$\begin{aligned}&-\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}=E\psi \qquad \text{as } x\neq 0\\ &\psi(0^+)=\psi(0^-)=\psi(0)=0\\ &\psi'(0^+)-\psi'(0)=-c\psi'(0).\end{aligned} \tag{2}$$

We can write $\psi=\theta(x)(a\,\mathrm{e}^{\mathrm{i}kx}+b\mathrm{e}^{-\mathrm{i}kx})+\theta(-x)(a'\,\mathrm{e}^{\mathrm{i}kx}+b'\,\mathrm{e}^{-\mathrm{i}kx})$, where $a$, $b$, $a'$ and $b'$ are constants to be determined, and $k^2=E$. From the boundary conditions in (2) we obtain, apart from a normalization constant,

$$\psi=\left[1-\frac{c}{2}\varepsilon(x)\right]\sin kx \tag{3}$$

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where $\varepsilon(x)=1$ as $x>0$, 0 as $x=0$, $-1$ as $x<0$, and we have taken $\theta(0)=\frac{1}{2}$ We can check the validity of (3) directly by the substitution of (3) into (1). We note that the singular term in $-\mathrm{d}^2\psi/\mathrm{d}x^2$ cancels the singular potential term.

In contrast with the above approach, in [2, 3] for one-centre $\delta'$-interaction,

$$\begin{aligned} &-\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}=E\psi \qquad \text{as } x\neq 0 \\ &\psi'(0^+)=\psi'(0^-) \\ &\psi(0^+)-\psi(0^-)=\beta\psi'(0) \end{aligned} \tag{4}$$

are proposed as the substitute for (1). From the discussion above, it is clear that the boundary conditions in (4) are irrelevant to the $\delta'$-interaction. Thus it is not surprising that if we substitute the eigenfunction in (4.23) of [3] into the Schrödinger equation with $\delta'$-interaction, we find that the eigenfunction does not satisfy the Schrödinger equation. Similar problems exist in the treatment of the many-centre $\delta'$-interaction in [2, 3].

We have noticed that the boundary conditions adopted by [4] are also irrelevant to the Schrödinger equation with $\delta'$-interaction.

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